ELEMENTS OF PSYCHOPHYSICS
Sections VII and XVI

Gustav Theodor Fechner (1860/1912)

Translated by Herbert Sidney Langfeld (1912)

[*Classics Editor's note: This translation of these passages from Fechner's* Elemente der Psychophysik *first appeared in Rand, Benjamin (Ed.)(1912).* The Classical Psychologists *(pp. 562-572). Boston: Houghton Mifflin, and was reprinted in Herrnstein, Richard J. & Boring, Edwin G. (Eds.)(1965).* A Source Book in the History of Psychology *(pp. 66-75). Cambridge, MA: Harvard University Press. The page numbers included her are from the Rand edition.*]

VII THE MEASUREMENT OF SENSATION

[...]

Weber's law, that equal relative increments of stimuli are proportional to equal increments of sensation, is, in consideration of its generality and the wide limits within which it is absolutely or approximately valid, to be considered fundamental for psychic measurement. There are, however, limits to its validity as well as complications, which we shall have carefully to examine later. Yet even where this law ceases to be valid or absolute, the principle of psychic measurement continues to hold, inasmuch as any other relation between constant increments of sensation and variable increments of stimulus, even though it is arrived at empirically and expressed by an empirical formula, may serve equally well as the fundamental basis for psychic measurement, and indeed must serve as such in those parts of the stimulus scale where Weber's law loses its validity. In fact such a law, as well as Weber's law, will furnish a differential formula from which may be derived an integral formula containing an expression for the measurement of sensation.

This is a fundamental point of view, *in which Weber's law, with its limitations, appears, not as limiting the application of psychic measurement, but as restricted in its own application toward that end and beyond which application the general* [p. 563] *principle of psychic measurement nevertheless continues to hold.* It is not that the principle depends for its validity upon Weber's law, but merely that the application of the law is involved in the principle.

Accordingly investigation in the interest of the greatest possible generalization of psychic measurement has not essentially to commence with the greatest possible generalization of Weber's law, which might easily produce the questionable inclination to generalize the law beyond its natural limitation, or which might call forth the objection that the law was generalized beyond these limits solely in the interest of psychic measurement; but rather it may quite freely be asked haw far Weber's law is applicable, and how far not; for the three methods which are used in psychic measurement are applicable even when Weber's law is not, and where these methods are applicable psychic measurement is possible.

In short, Weber's law forms merely the basis for the most numerous and important applications of psychic measurement, but not the universal and essential one. The most general and more fundamental basis for psychic measurement is rather those methods by which the relation between stimulus increments and sensation increment in general is determined, within, as well as without, the limits of Weber's law; and the development of these methods towards even greater precision and perfection is the most important consideration in regard to psychic measurement.

And yet a great advantage would be lost, if so simple a law as Weber's law could not be used as an exact or at least sufficiently approximate basis for psychic measurement; just such an advantage as would be lost if we could not use the Kepler law in astronomy, or the laws of simple refraction in the theory of the dioptric instruments. Now there is just the same difficulty with these laws as with Weber's law. In the case of Kepler's law we abstract from deviations. In the case of simple lens refraction we abstract from optical aberration. In fact they may become invalid as soon as the simple hypotheses for which they are true no longer exist. Yet they will always remain [p. 564] decisive for the principle relation with which astronomy and dioptrics are concerned. Weber's law may in like manner, entirely lose its validity, as soon as the average or normal conditions under which the stimulus produces the sensation are unrealized. It will always, however, be decisive for these particular conditions.

Further, just as in physics and astronomy, so can we also in psychic measurement, neglect at first the irregularities and small departures from the law in order to discover and examine the principle relations with which the science has to do. The existence of these exceptions must not, however, be forgotten, inasmuch as the finer development and further progress of the science depends upon the determination and calculation of them, as soon as the possibility of doing so is given.

The determination of psychic measurement is a matter for outer psychophysics and its first applications lie within its boundary; its further applications and consequences, however, extend necessarily into the domain of inner psychophysics and its deeper meaning lies there. It must be remembered that the stimulus does not cause sensation directly, but rather through the assistance of bodily processes with which it stands in more direct connection. The dependence, quantitatively considered of sensation on stimulus, must finally be translated into one of sensation on the bodily processes which directly underlie the sensation -- in short the psycho-physical processes; and the sensation, instead of being measured by the amount of the stimulus, will be measured by the intensity of these processes. In order to do this, the relation of the inner process to the stimulus must be known. Inasmuch as this is not a matter of direct experience it must be deduced by some exact method. Indeed it is possible for this entire investigation to proceed along exact lines, and it cannot fail at some time or other to obtain the success of a critical study, if one has not already reached that goal.

Although Weber's law, as applied to the relation of stimulus to sensation, shows only a limited validity in the domain of outer psychophysics, it has, as applied to the relation of sensation [p. 565] to kinetic energy, or as referred to some other function of the psycho-physical process, in all probability an unlimited validity in the domain of inner psychophysics, in that all exceptions to the law which we find in the arousal of sensation by external stimulus, are probably due to the fact that the stimulus only under normal or average conditions engenders a kinetic energy in those inner processes proportional to its own amount. From this it may be foreseen, that this law, after it has been restated as a relation between sensation and the psycho-physical processes, will be as important, general, and fundamental for the relations of mind to body, as is the law of gravity for the field of planetary motion. And it also has that simplicity which we are accustomed to find in fundamental laws of nature.

Although, then, psychic measurement depends upon Weber's law only within certain limitations in the domain of outer psycho-physics, it may well get its unconditional support from this law in the field of inner psychophysics. These are nevertheless for the present merely opinions and expectations, the verification of which lies in the future.

XVI.[*] THE FUNDAMENTAL FORMULA AND THE MEASUREMENT FORMULA

Although not as yet having a measurement for sensation, still one can combine in an exact formula the relation expressed in Weber's law, -- that the sensation difference remains constant when the relative stimulus difference remains constant, -- with the law, established by the mathematical auxiliary principle, that small sensation increments are proportional to stimulus increments. Let us suppose, as has generally been done in the attempts to preserve Weber's law, that the difference between two stimuli, or, what is the same, the increase in one stimulus, is very small in proportion to the stimulus itself. Let the stimulus which is increased be called β, the small increase *d*β, where the letter *d* is to be considered not as a special magnitude, but simply as a sign that *d*β is the small increment of [p. 566] β. This already suggests the differential sign. The relative stimulus increase therefore is *d*β/β. On the other hand, let the sensation which is dependent upon the stimulus β be called *d*γ, and let the small increment of the sensation which results from the increase of the stimulus by *d*β be called *d*γ, where *d* again simply expresses the small increment. The terms dβ and dγ are each to be considered as referring to an arbitrary unit of their own nature.

According to the empirical Weber's law, *d*γ remains constant when dβ/β remains constant, no matter what absolute values *d*β and β take; and according to the *a priori* mathematical auxiliary principle the changes *d*γ and *d*β remain proportional to one another so long as they remain very small. The two relations may be expressed together in the following equation:

$$d\gamma = K d\beta/\beta \qquad (1)$$

where κ [1] is a constant (dependent upon the units selected for γ and β). In fact, if one multiplies β*d* and β by any number, so long as it is the same number for both, the proportion remains constant, and with it also the sensation difference *d*γ. This is Weber's law. If one doubles or triples the value of the variation *d*β without changing the initial value β, then the value of the change *d*γ is also doubled or tripled. This is the mathematical principle. The equation dγ = *Kd*β/β therefore entirely satisfies both Weber's law and this principle; and no other equation satisfies both together. This is to be called the *fundamental formula*, in that the deduction of all consequent formulas will be based upon it.

The fundamental formula does not presuppose the measurement of sensation, nor does it establish any; it simply expresses the relation holding between small relative stimulus increments and sensation increments. In short, it is nothing more than Weber's law and the mathematical auxiliary principle united and expressed in mathematical symbols. [p. 567]

There is, however, another formula connected with this formula by infinitesimal calculus, which expresses a general quantitative relation between the stimulus magnitude as a summation of stimulus increments, and the sensation magnitude as a summation of sensation increments, in such a way, that with the validity of the first formula, together with the assumption of the fact of limen, the validity of this latter formula is also given.

Reserving for the future a more exact deduction, I shall attempt first to make clear in a general way the connection of the two formulas.

One can readily see, that the relation between the increments dγ and dβ in the fundamental formula corresponds to the relation between the increments of a logarithm and the increments of the corresponding number. For as one can easily convince oneself, either from theory or from the table, the logarithm does not increase by equal increments when the corresponding number increases by equal increments,. but rather when the latter increases by equal relative amounts; in other words, the increases in the logarithms remain equal, when the relative increases of the numbers remain equal. Thus, for example, the following numbers and logarithms belong together:

Number.	Logarithm.

10	1.000000
11	1.0413927
100	2.000000
110	2.0413927
1000	3.000000
1100	3.0413927

where an increase of the number 10 by 1 brings with it just great an increase in the corresponding logarithm, as the increase of the number 100 by 10 or 1000 by 100. In each instance the increase in the logarithm is 0.0413927. Further, as was already shown in explaining the mathematical auxiliary principle, the increases in the logarithms are proportional to the increases of [p. 568] the numbers, so long as they remain very small. Therefore one can say, that Weber's law and the mathematical auxiliary principle are just as valid for the increases of logarithms and numbers in their relation to one another, as they are for the increases of sensation and stimulus.

The fact of the threshold appears just as much in the relation of a logarithm to its number as in the relation of sensation to stimulus. The sensation begins with values above zero, not with zero, but with a finite value of the stimulus -- the threshold; and so does the logarithm begin with values above zero, not with a zero value of the number, but with a finite value of the number, the value I, inasmuch as the logarithm of 1 is equal to zero.

If now, as was shown above, the increase of sensation and stimulus stands in a relation similar to that of the increase of logarithm and number, and, the point at which the sensation begins to assume a noticeable value stands in a relation to the stimulus similar to that which the point at which the logarithm attains positive value stands to the number, then one may also expect that sensation and stimulus themselves stand in a relation to one another similar to that of logarithm to number, which, just as the former (sensation and stimulus) may be regarded as made up of a sum of successive increments.

Accordingly the simplest relation between the two that we can write is $\gamma = \log\beta$.

In fact it will soon be shown that, provided suitable units of sensation and stimulus are chosen, the functional relation between both reduces to this very simple formula. Meanwhile it is not the most general formula that can be derived, but one which is only valid under the supposition of particular units of sensation and stimulus, and we still need a direct and absolute deduction instead of the indirect and approximate one.

The specialist sees at once how this may be attained, namely, by treating the fundamental formula as a differential formula and integrating it. In the following chapter one will find this done. Here it must be supposed already carried out, and those who are not able *to* follow the simple infinitesimal deduction, [p. 569] must be asked to consider the result as a mathematical fact. This result is the following functional formula between stimulus and sensation, which goes by the name of the measurement formula and which will now be further discussed:

$$\gamma = \kappa(\log\beta - \log b) \qquad (2)$$

In this formula κ again stands for a constant, dependent upon the unit selected and also the logarithmic system, and *b* a second constant which stands for the threshold value of the stimulus, at which the sensation γ begins and disappears.

According to the rule, that the logarithm of a quotient of two numbers may be substituted for the difference of their logarithms, ... one can substitute for the above form of the measurement formula the following, which is more convenient for making deductions.

$$\gamma = \kappa(\log \beta/b) \qquad (3)$$

From this equation it follows that the sensation magnitude γ is not to be considered as a simple function of the stimulus value β, but of its relation to the threshold value *b*, where the sensation begins and disappears. This relative stimulus value, β/*b* is for the future to be called the fundamental stimulus value, or the fundamental value of the stimulus.

Translated in words, the measurement formula reads:

The magnitude of the sensation (γ) *is not proportional to the absolute value of the stimulus* (β)*, but rather to the logarithm of the magnitude of the stimulus, when this last is expressed in terms of its threshold value(b), i.e. that magnitude considered as unit at which the sensation begins and disappears. In short, it is proportional to the logarithm of the fundamental stimulus value.*

Before we proceed further, let us hasten to show that that relation between stimulus and sensation, from which the measurement formula is derived, may be correctly deduced in turn from it, and that this latter thus finds its verification in so far as these relations are found empirically. We have here at the same time the simplest examples of the application of the measurement formula. [p. 570]

The measurement formula is founded upon Weber's law and the fact of the stimulus threshold; and both must follow in turn from it.

Now as to Weber's law. In the form that equal increments of sensation are proportional to relative stimulus increments, it may be obtained by differentiating the measurement formula, inasmuch as in this way one returns to the fundamental formula, which contains the expression of the law in this form. In the form, that equal sensation differences correspond to equal relations of stimulus, the law may be deduced in quite an elementary manner as follows.

Let two sensations, whose difference is to be considered, be called γ and γ', and the corresponding stimuli β and β'. Then according to the measurement formula

$$\gamma = \kappa(\log\beta - \log b)$$

$$\gamma = \kappa(\log\beta' - \log b)$$

and likewise for the sensation difference

$$\gamma - \gamma' = \kappa(\log\beta - \log\beta')$$

or, since $\log\beta - \log\beta' = \log\beta/\beta'$

$$\gamma - \gamma' = \kappa(\log\beta/\beta')$$

From this formula it follows, that the sensation difference $\gamma - \gamma'$ is a function of the stimulus relation β/β', and remains the same no matter what values β, β' may take, so long as the relation remains unchanged, which is the statement of Weber's law.

In a later chapter we shall return to the above formula under the name of the difference formula, as one of the simplest consequences of the measurement formula. As for the fact of the threshold, which is caused by the sensation having zero value not at zero but at a finite value of the stimulus, from which point it first begins to obtain noticeable values with increasing values of stimulus, it is so far contained in the measurement formula as γ does not, according to this formula, have the value zero when $\beta=0$, but when β is equal to a finite value *b*. This follows as well from equation [p. 571] (2) as (3) of the measurement formula, directly from (2), and from (3) with the additional consideration of the fact, that when β equals *b*, $\log \beta/b$ equals log1, and log1=0.

Naturally all deduction from Weber's law and the fact of the threshold will also be deductions from our measurement formula.

It follows from the former law, that every given increment of stimulus causes an ever decreasing increment in sensation in proportion as the stimulus grows larger, and that at high values of the stimulus it is no longer sensed, while on the other hand, at low values it may appear exceptionally strong.

In fact the increase of a large number β by a given amount is accompanied by a considerably smaller increase in the corresponding logarithm γ, than the increase of a small number β by the same amount. When the number 10 is increased by 10, (that is, reaches 20), the logarithm corresponding to 10, which is 1, is increased to 1.3010. When, however, the number 1000 is increased by 10, the logarithm corresponding to 1000, namely 3, is only increased to 3.0043· In the first case the logarithm is increased by 1-3 [one third]of its amount, in the latter case by about 1-700 [one seven-hundredth].

In connection with the fact of the threshold belongs the deduction, that a sensation is further from the perception threshold the more the stimulus sinks under its threshold value. This distance of a sensation from the threshold, is represented in the same manner by the negative values of γ, according to our measurement formula, as the increase above the threshold is represented by the positive values.

In fact one sees directly from equation (2), that when β is smaller than *b* and with it $\log\beta$ smaller than $\log b$, the sensation takes on negative values, and the same deduction follows in equation (3), in that β/b' becomes a proper fraction when $\beta<b$, and the logarithm of a proper fraction is negative.

In so far as sensations, which are caused by a stimulus which is not sufficient to raise them to consciousness, are called unconscious, and those which affect consciousness are called [p. 572] conscious, we may say that the unconscious sensations are represented in our formula by negative, the conscious by positive values. We will return to this statement in a special chapter (chapter 18) since it is of great importance, and perhaps not directly evident to everyone. For the present I shall not let it detain me longer.

According to the foregoing our measurement formula corresponds to experience:

1. In the cases of equality, where a sensation difference remains the same when the absolute intensity of the stimulus is altered (Weber's law).

2. In the cases of the thresholds, where the sensation itself ceases, and where its change becomes either

imperceptible or barely perceptible. In the former case, when the sensation reaches its lower threshold; in the latter case, when it becomes so great that a given stimulus increase is barely noticed.

3. In the contrasting cases, between sensations which rise above the threshold of consciousness and those that do not reach it, -- in short, conscious and unconscious sensations.

From the above the measurement formula may be considered well founded.

In the measurement formula one has a general dependent relation between the size of the fundamental stimulus and the size of the corresponding sensation and not one which is valid only for the cases of equal sensations. This permits the amount of sensation to be calculated from the relative amounts of the fundamental stimulus and thus we have a measurement of sensation.

Footnote

[1] *Classics Editor's note: Fechner uses K in the equation, but kappa here. He also uses K in the equation later in this same paragraph, but kappa repeatedly later in the section.*

[*]Classics Editor's note: In the Rand edition, this chapter is incorrectly cited as "XIV" instead of "XVI." It is correctly cited in the Herrnstein & Boring edition.